Hall effect measurements for determining the band gap energy of undoped germanium, including the conductivity, charge carrier type, concentration and mobility for n-type and p-type doped germanium

Hassan Mirza[1] and Danny Morris[1]
[1]School of Physics and Astronomy, Queen Mary, University of London, London E1 4NS, England

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In this experiment, the band gap energy of undoped germanium was measured and found to be 0.73 ± 0.03 eV. This is in agreement with and very close to the literary value of 0.67 eV. The Hall coefficients of n-type and p-type doped germanium samples were measured and found to be $R_{H,n} = (8.6 \pm 0.2) \times 10^{-3}\ \mathrm{m^3C^{-1}}$ and $R_{H,p} = (8.0 \pm 0.2) \times 10^{-3}\ \mathrm{m^3C^{-1}}$ respectively. Next, the conductivities of the doped germanium samples were measured, and found to be $\sigma_{0,n} = 43.25 \pm 1.31\ \mathrm{Sm^{-1}}$ and $\sigma_{0,p} = 37.31 \pm 1.19\ \mathrm{Sm^{-1}}$ for n-type and p-type respectively. The charge carrier mobilities for the samples were also measured, and the calculated values were $\mu_{H,n} = (3.72 \pm 0.61) \times 10^{-5}\ \mathrm{cm^2V^{-1}s^{-1}}$ for the n-type sample, and $\mu_{H,p} = (2.98 \pm 0.49) \times 10^{-5}\ \mathrm{cm^2V^{-1}s^{-1}}$ for the p-type sample. Lastly, the carrier concentrations were found, and these were $n_n = (7.27 \pm 0.29) \times 10^{14}\ \mathrm{cm^{-3}}$ and $n_p = (7.81 \pm 0.33) \times 10^{14}\ \mathrm{cm^{-3}}$ respectively. The charge carriers were found to be electrons for the n-type sample, and holes for the p-type.

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I. INTRODUCTION

The Hall effect was first demonstrated by Edwin Hall in 1879. It is the name given to the production of a voltage difference (Hall voltage) within an electrical conductor through the effect of an applied magnetic field. In this experiment, Hall measurements were made to:

1. determine the band gap energy of an undoped sample of germanium by measuring its conductivity as a function of temperature,
2. measure the Hall voltage as a function of current, magnetic field and temperature of n-type (excess holes) and p-type (excess free electrons) germanium samples that have been doped (i.e. impurity atoms have been added), and
3. determine the type of charge carrier (electrons or holes) and calculate the carrier concentration and carrier mobility for both of the n- and p-type samples by finding the Hall coefficient.

II. THEORETICAL BACKGROUND

A. The Hall effect

The flow of electrons and holes (current) within a conductor changes as a result of applying an external magnetic field. In the absence of such a field, current flows in a relatively straight path between collisions across the terminals of the conductor.[1] However, when a magnetic field perpendicular to the direction of current flow is applied, the straight paths between collisions become curved; as the charge carriers now experience what is known as the Lorentz force, which can be expressed as

$$\vec{F} = e(\vec{v} \times \vec{B}) \tag{1}$$

where e is the elementary charge, $\vec{v}$ is the charge carrier velocity and $\vec{B}$ is the magnetic field. The Lorentz force deflects the charge carriers towards one end of the conductor, while equal and opposite charge gathers at the opposite end. Charges continue to accumulate at the ends until an electric field is generated so as to cancel out the magnetic field. At this point, charge begins to flow in a straight line, giving rise to the Hall voltage, as shown in Fig. 1:

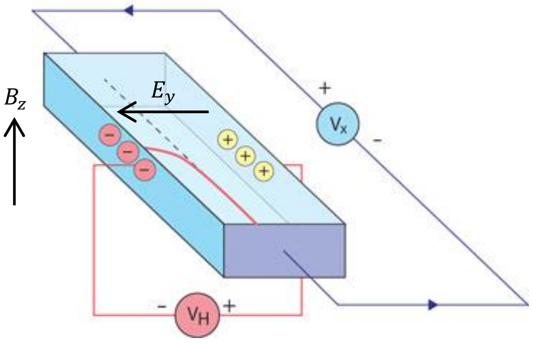

FIG. 1. The application of an external magnetic field, B_z, perpendicular to the direction of charge flow, causes negative charges to gather on the left side and positive charges on the right. This induces an electric field E_y which balances B_z. The voltage across the sides is the Hall voltage, V_H, which is clearly different from the sample voltage, V_x, measured across the length of the sample.

B. Conductivity and band gap energy

1. *Conductivity of a semiconductor sample*

The electrical conductivity of a material is given by

$$\sigma = \frac{1}{\rho} = \frac{l}{A}\frac{I}{V} \tag{2}$$

where ρ is the resistivity, l is the length of the sample, A is the cross sectional area, I is the current and V is the voltage. The doped and undoped germanium samples used in this experiment were 20 mm long, 10 mm high and 1 mm thick.

To determine the conductivity of an undoped sample,[2] σ_0, through measurements of voltage, we require the value of voltage across the sample when its magnetic field strength is at a minimum. This is achieved by plotting a graph of sample voltage as a function of magnetic field strength, (which is expected to be parabolic in shape), and then reading off the required voltage value, V_0. This is the value that corresponds to the point on the voltage axis where the magnetic field strength axis is 0. Provided that the current I remains fixed throughout the course of the experiment, we may use Ohm's law, which states

$$V_0 = IR_0 \tag{3}$$

in transposed form (for the target R_0) to first find the sample's resistance; which can then be used further in the following expression,

$$\sigma_0 = \frac{l}{R_0 A} \tag{4}$$

where l is the length of the sample and A is the area of cross section, to obtain its conductivity.

2. *Band gap energy*

Semiconductors have conductivities that depend on temperature, as shown in Fig. 2. Starting at low temperatures (close to ambient), there are three phases of such dependence for a doped material sample: (I) extrinsic conduction, (II) impurity depletion and (III) intrinsic conduction.[3] In phase (I), increases in temperature cause a linear increase in conductivity. This is due to charge carriers being released from the impurities within the sample. The second phase (II) begins at moderate temperatures once all the impurities have been activated – further increases in temperature no longer cause an increase in conductivity. In the third phase (III), which occurs at high temperatures, charge carriers are created by transitioning from the valence band into the conduction band due to thermal excitation, as shown in Fig. 3.

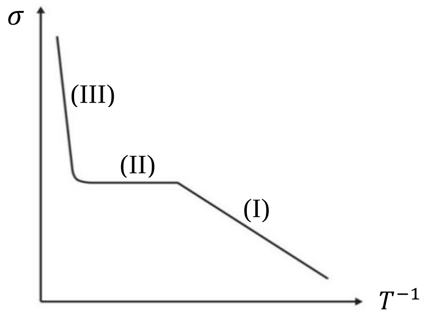

FIG. 2. A theoretical graph of conductivity as a function of reciprocal temperature for an ideal doped semiconductor sample. In phase (I), conductivity increases with temperature due to charge carriers being produced from impurity atoms within the sample. Phase (II) is when there is no increase in conductivity with increasing temperature. Finally, at high temperatures in phase (III), the conduction band begins to populate as charge carriers transition from the valence band due to thermal excitation.

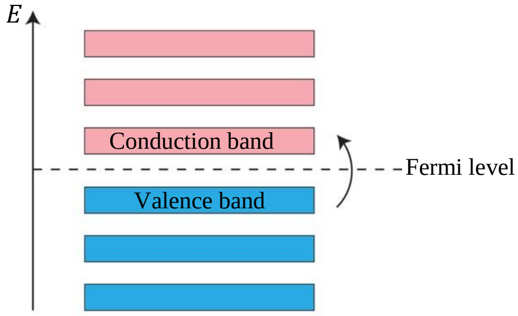

FIG. 3. In semiconductors, charge carriers transition from the valence band to the conduction band due to thermal excitation, which occurs at high temperatures.

For phase (III), the temperature dependence of conductivity can be modelled as an exponential function

$$\sigma = \sigma_0 e^{-\frac{E_g}{2k_B T}} \tag{5}$$

where E_g is the band gap energy, k_B is the Boltzmann constant and T is the absolute temperature. It is clear that a plot of $\ln(\sigma)$ as a function of T^{-1} will yield a straight-line graph with a gradient given by

$$b = -\frac{E_g}{2k_B} \tag{6}$$

from which the band gap energy E_g can be calculated. An undoped sample's conductivity will only increase through valence-conduction band transitions [phase (III)] as there are no impurities to produce charge carriers. This makes it a better choice of sample for determining the band gap energy, as opposed to a doped sample.

C. Hall coefficient, carrier concentration and carrier mobility

The carrier concentration n and carrier mobility μ_H are related by the Hall coefficient, R_H. This coefficient is the gradient of a graph of Hall voltage as a function of magnetic flux density:

$$R_H = \frac{V_H}{B}\frac{d}{I} \tag{7}$$

where d is the depth of the sample (1 mm), I is the supplied current and B is the varying magnetic field strength. The carrier mobility and concentration, respectively, are then given by

$$\mu_H = R_H \sigma_0 \tag{8}$$

and

$$n = \frac{1}{eR_H} \tag{9}$$

which may be determined once the conductivity and Hall coefficient of the sample in question have been found.

III. EXPERIMENTAL SETUP AND PROCEDURE

A. The band gap energy of (undoped) germanium

An undoped germanium sample was placed into a Hall effect module (HEM) connected to a 12 V AC power supply. A digital multimeter was connected across the sample (for measuring the sample voltage) using the lower set of sockets. On the HEM unit, the measurement mode was toggled and set to display the current (in mA) through the sample. This was set to 5 ± 1 mA before switching the display back to temperature (in ^{O}C) mode. At this point, the temperature was increased to the maximum possible value the apparatus could reach, which was 160 ± 1 ^{O}C. As the temperature decreased, measurements of the corresponding sample voltages, in steps of 5 ^{O}C, were read off of the multimeter display and recorded.

B. Hall measurements of n-type and p-type samples of doped germanium

1. Initial setup of the apparatus

The undoped sample in the HEM was replaced with a doped n-type sample. The multimeter was set to read voltages to the highest precision (± 0.001 V, as displayed on the LED screen of the multimeter). The magnet connected to the teslameter was placed around the germanium sample for measuring its magnetic field strength. To ensure that the magnetic field is measured on the germanium sample directly, the teslameter was set to read 0 mT while the Hall probe was not inside the HEM.

2. Measuring Hall voltage as a function of current

In this series of measurements, the magnetic field strength was kept fixed at 250 ± 5 mT by altering the current and voltage delivered by the power supply. The multimetre was connected to the upper set of sockets on the HEM which measure the Hall voltage. Setting the display on the HEM to read current in mA enabled us to measure the Hall voltage as the current was varied on the HEM from -60 to +60 mA, in steps of about 5 ± 1 mA.

3. Measuring sample voltage as a function of magnetic field strength

The current on the HEM was set to 25 ± 1 mA and the multimeter was connected to the lower set of sockets for measuring sample voltage. Using the teslameter, the magnetic field was varied from 0 to 300 mT in steps of 20 ± 5 mT while the corresponding sample voltages displayed on the multimeter were recorded.

4. Measuring Hall voltage as a function of magnetic field strength

Measurements for this part of the experiment were made by setting the current on the HEM to a fixed value of 30 ± 1 mA. The multimeter is connected to measure the Hall voltage of the sample. This measurement required the polarity of the coil-current to be reversed, which was achieved by switching the order in which the cables are connected in the sockets of HEM. The magnetic field strength was varied from -300 to +300 mT in this way by reversing the polarity at 0 mT. The measurements were taken in steps of 20 ± 5 mT and the corresponding Hall voltages were recorded.

5. Measuring Hall voltage as a function of sample temperature

Setting a constant value of current at 30 mA and the magnetic field strength at 300 mT, the Hall probe was removed from the HEM and the temperature increased to the maximum value (170 ± 1 ^{O}C). Hall voltages were recorded in steps of 10 ± 1 ^{O}C as the temperature dropped.

The same measurements were made for the p-type doped germanium sample.

IV. EXPERIMENTAL RESULTS AND ANALYSIS

A. Result for the band gap energy of undoped germanium

Since the germanium sample used in this part of the experiment was undoped, the data collected represented a well-defined straight line similar to the one corresponding to phase (III) in Fig. 2 when transformed to fit the scales of the axes. From the graph shown in Fig. 4 (see Appendix for the source data of all parts of the experiment), the gradient value obtained was

$$b = -4228.2 \pm 6.1$$

Substituting this into Eq. (4) and transposing for the target E_g, we obtain the band gap energy for germanium

$$E_g = (-4228.2)(-2k_B)$$

$$E_g = 0.73 \pm 0.03 \text{ eV}$$

Comparing this with the literary value of 0.67 eV, we see that the experimentally calculated value is well in agreement as it is less than one standard deviation out. This is due to how well the data fits the theoretical model of phase (III) shown in Fig. 2.

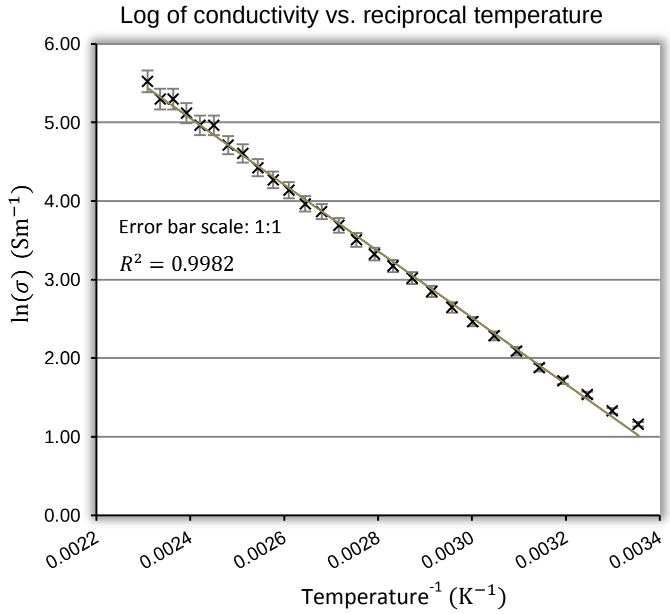

FIG. 4. A graph of $\ln(\sigma)$ as a function of K^{-1}; for finding the valence-conduction band gap energy of undoped germanium. The relationship is strongly linear as the coefficient of determination is close to unity. The line fits well with the theoretical model of phase (III) shown in fig. 2.

B. Results for the Hall measurements of n-type and p-type samples of doped germanium

The experimental procedure was carried out for both the n-type and p-type respectively. The main objective for these measurements was to determine the type of charge carriers, their concentration and mobility.

1. *Results for the measurements of Hall voltage as a function of current*

This part of the experiment was to demonstrate the linear relationship between the Hall voltage and current of the undoped samples of germanium. As can be seen from Fig. 5, the strong linear relationship between current and Hall voltage is clear for both samples of germanium.

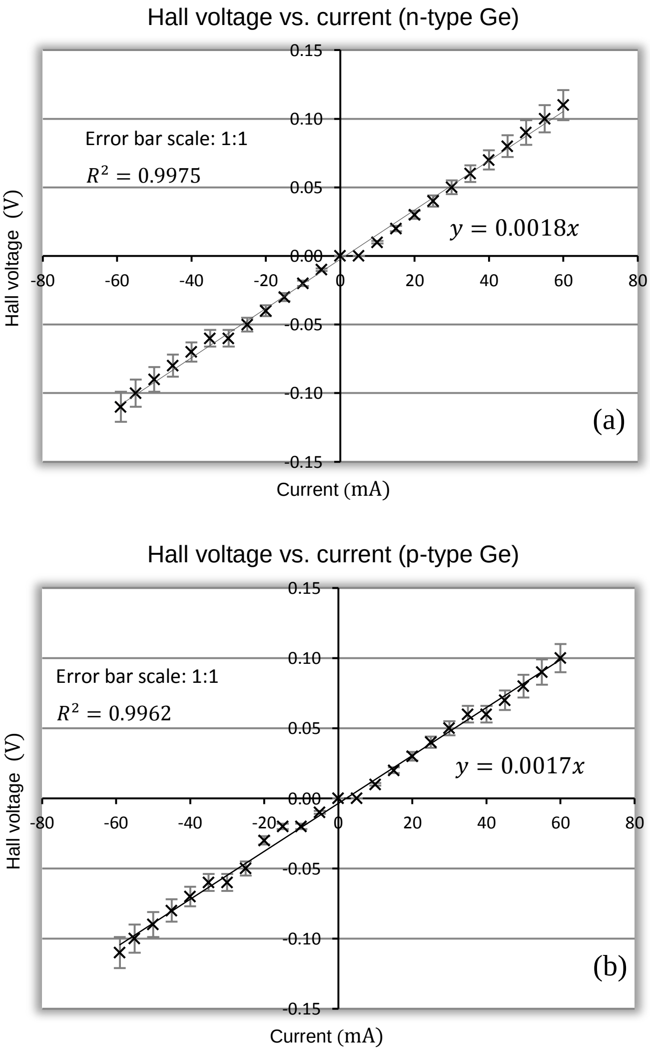

FIG. 5. The graphs for Hall voltage as a function of current, for both the n-type and p-type germanium samples respectively. In both cases, the linear relationship between the quantities is clear.

2. *Measuring sample voltage as a function of magnetic field strength*

The next set of measurements were to determine the intrinsic resistances, R_0, for both the germanium samples. The intrinsic resistance is needed for calculating the conductivities of the samples.

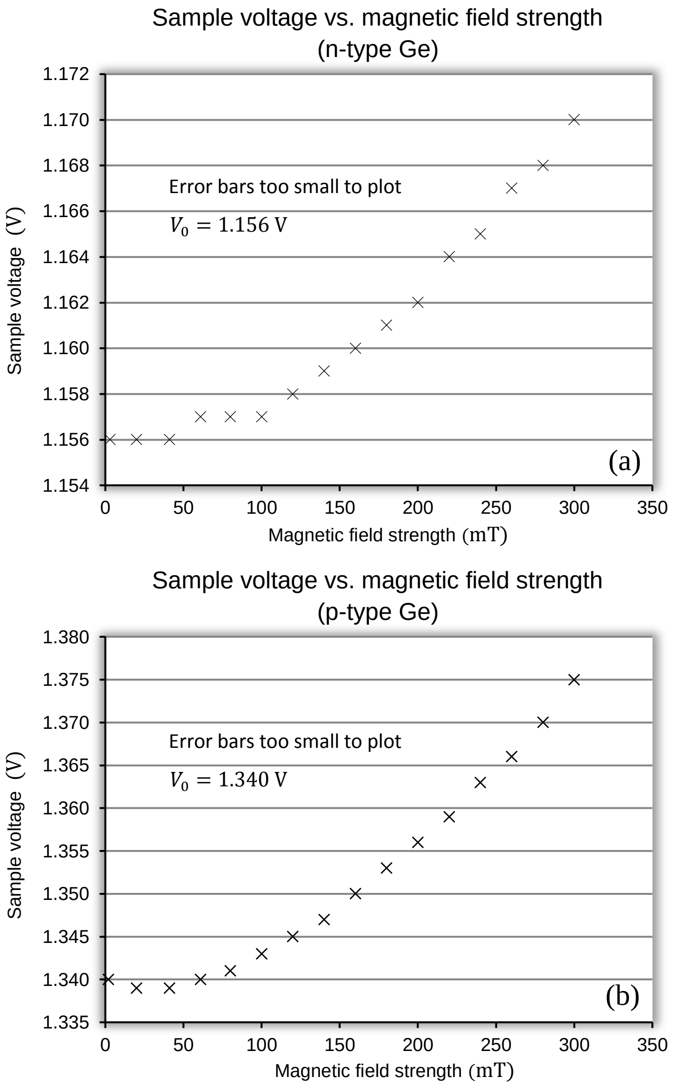

FIG. 6. The graphs for sample voltage as a function of magnetic field strength, for both the n-type and p-type germanium samples respectively. As expected, a quadratic relationship between the two quantities is clear in both graphs, although it is more evident for the p-type sample.

To determine the intrinsic resistances, we make use of the fact that the current supplied to the samples in this part of the experiment is fixed at 25 mA. From the graphs shown in Fig. 6, we obtained the values of the voltage, V_0, across each sample when the magnetic field strength is minimum. Using Eq. (3), the intrinsic resistances are found as follows:

For the n-type and p-type samples respectively, the intrinsic resistances are

$$R_{0,n} = \frac{1.156}{0.025} = 46.24 \pm 0.17\ \Omega$$

and

$$R_{0,p} = \frac{1.134}{0.025} = 53.60 \pm 0.23\ \Omega$$

Once the gradient of the graph of Hall voltage as a function of magnetic field strength has been determined, the values of carrier concentration and mobility can be found.

3. *Results for the measurements of Hall voltage as a function of magnetic field strength*

The data collected for the Hall voltage as a function of magnetic field strength show a strong linear relationship between the two quantities for both samples, as shown in Fig. 7.

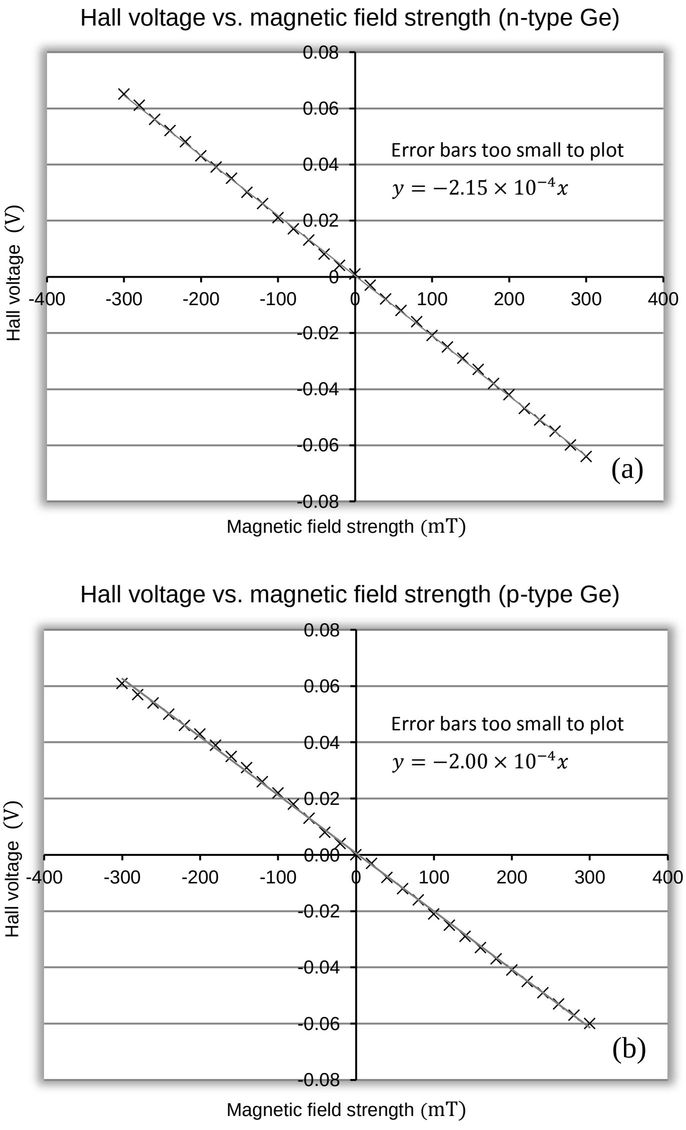

FIG. 7. The graphs for Hall voltage as a function of magnetic field strength for both the n-type and p-type germanium.

We can now find the Hall coefficient, conductivity, carrier mobility and concentration for each sample:

a. For n-type germanium. The gradient is given in terms of mT. The magnitude of this is then used in Eq. (7) to find R_H. Thus, using the value of fixed current at 0.025 A and the depth of the sample 0.001 m; the hall coefficient is

$$R_H = \frac{V_H}{B}\frac{d}{I} = 2.15 \times 10^{-4} \times 1000 \times \frac{0.001}{0.025}$$
$$= (8.6 \pm 0.2) \times 10^{-3}\ \mathrm{m^3C^{-1}}$$

and the conductivity of the sample, using Eq. (4) with the length equal to 0.02 m, the cross sectional area equal to $10^{-5}\ \mathrm{m^2}$ and the intrinsic resistance equal to $46.24\ \Omega$, is equal to:

$$\sigma_0 = \frac{l}{R_0 A} = \frac{0.02}{46.24 \times 10^{-5}}$$
$$= 43.25 \pm 1.31\ \mathrm{Sm^{-1}}$$

Similarly, the carrier mobility can be found using Eq. (8):

$$\mu_H = R_H\sigma_0 = 8.6 \times 10^{-3} \times 43.25$$
$$\mu_H = (3.72 \times \pm 0.61)10^{-5}\ \mathrm{cm^2V^{-1}s^{-1}}$$

And, finally, the carrier concentration can be found using Eq. (9):

$$n = \frac{1}{1.6 \times 10^{-19} \times 8.6 \times 10^{-3}}$$
$$= (7.27 \pm 0.29) \times 10^{14}\ \mathrm{cm^{-3}}$$

b. For p-type germanium. Following similar lines of reasoning, we obtain the same parameters for the p-type sample:

$$R_H = (8.0 \pm 0.2) \times 10^{-3}\ \mathrm{m^3C^{-1}}$$
$$\sigma_0 = 37.31 \pm 1.19\ \mathrm{Sm^{-1}}$$
$$\mu_H = (2.98 \pm 0.49) \times 10^{-5}\ \mathrm{cm^2V^{-1}s^{-1}}$$
$$n = (7.81 \pm 0.33) \times 10^{14}\ \mathrm{cm^{-3}}$$

4. Results for measurements of Hall voltage as a function of sample temperature

To determine the type of charge carriers, we must observe the properties of a graph of Hall voltage as a function of temperature for both samples. Electrons and holes initially travel in opposite directions through the sample, but the application of a perpendicular magnetic field causes both charge carriers to be deflected in the same direction – if the directions of the current and magnetic field are known, the type of charge carrier can be deduced by the sign of the Hall voltage.[4] If the sign is negative, the charge carriers causing the flow of current are holes whereas if the sign is positive, the charge carriers causing the flow of current are electrons.

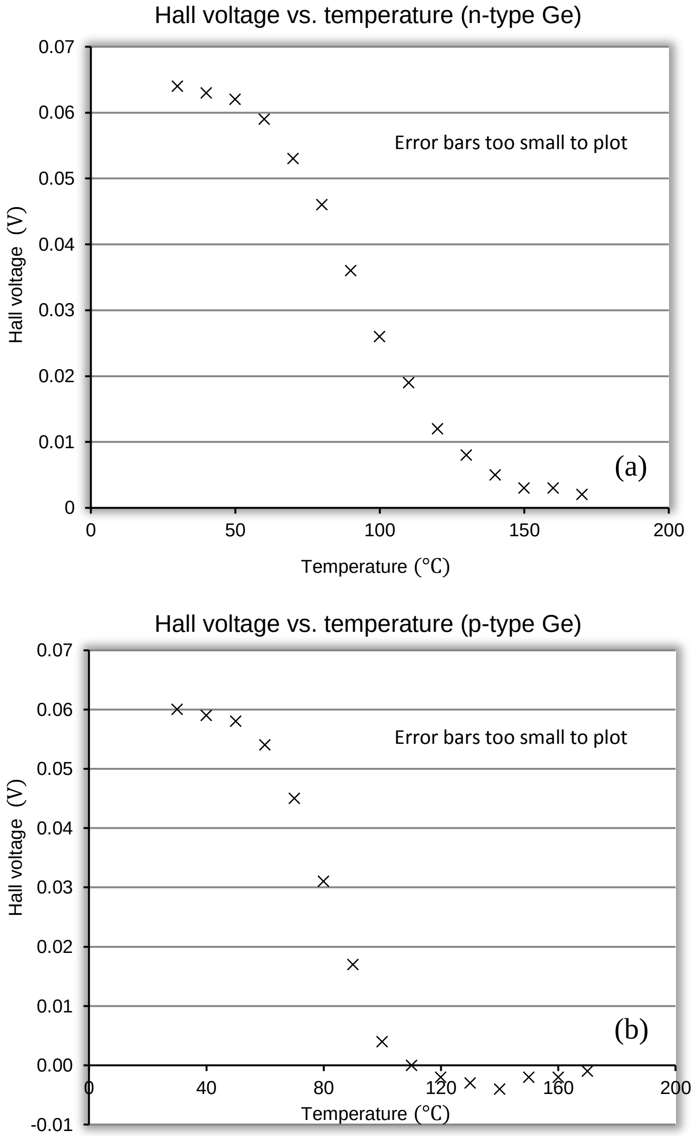

FIG. 8. The graphs for Hall voltage as a function of sample temperature. In Fig. 8(b), we see that the Hall voltage changes sign at about 110 ^{O}C. This is characteristic of p-type materials in general beyond a certain temperature.

From Fig. 8(a), it is clear that there is no change in sign of the Hall voltage for the entire domain of temperature. It can thus be deduced that the type of charge carriers responsible for current flow in n-type germanium is electrons. This is to be expected as n-type dopants have more valence electrons than the base to which they are added. On the other hand, in Fig. 8(b), we observe a typical property of p-type doped materials; that beyond a certain temperature, the polarity of the Hall voltage reverses. This shows that the that, for p-type doped germanium, the charge carriers causing the flow of current are holes.

V. ERRORS

Systematic errors in this experiment were mainly due to the fluctuations of readings on the digital devices, along with the values not being 0 when required (which is most likely due to poor calibration). For example, the sample voltage should be 0 when there is no applied magnetic field. This was not the case, but it was corrected by forcing the readings to 0 on the apparatus.

VI. CONCLUSION

We have found that the band gap energy for undoped germanium is 0.73 ± 0.3 eV. For undoped n- and p-type germanium, we also calculated the conductivity, Hall coefficient, Hall mobility and concentration of charge carriers for both n- and p-type germanium, as summarized in Table IV. These calculations were made by plotting graphs using data on both samples' Hall voltages, magnetic field strength, current and temperature.

The type of charge carriers responsible for current in n-type and p-type doped germanium were also determined. For the former, it was deduced that the charge carriers are electrons whereas for the latter, it was holes. This is consistent with the fact that n-type dopants have excess valence electrons while p-type dopants have excess holes.

Table VI. The experimentally determined values of Hall coefficients, conductivities, Hall mobilities and carrier concentrations for n-type and p-type doped germanium.

Sample	Hall coefficient $\mathrm{m^3C^{-1}}$	Conductivity $\mathrm{Sm^{-1}}$	Hall mobility $\mathrm{cm^2V^{-1}s^{-1}}$	Carrier concentration $\mathrm{cm^{-3}}$
n-type Ge	$(8.6 \pm 0.2) \times 10^{-3}$	43.25 ± 1.31	$(3.72 \pm 0.61) \times 10^{-5}$	$(7.27 \pm 0.29) \times 10^{14}$
p-type Ge	$(8.0 \pm 0.2) \times 10^{-3}$	37.31 ± 1.19	$(2.98 \pm 0.49) \times 10^{-5}$	$(7.81 \pm 0.33) \times 10^{14}$

APPENDIX

Table I. Source data for Fig. 4.

Temperature $\pm 1\ ^{\mathrm{O}}\mathrm{C}$	Voltage $\pm 0.01\ \mathrm{V}$	Absolute Temperature $\pm 2.5\%\ \mathrm{K}$	Absolute Temperature^{-1} $\pm 2.5\%\ \mathrm{K}^{-1}$	Voltage^{-1} $\pm 0.25\%\ \mathrm{V}$	Conductivity $\pm 2.5\%\ \mathrm{Sm}^{-1}$	$\ln \sigma\ \pm 2.5\%$
160	0.04	433.15	0.00231	25.00	250.00	5.52
155	0.05	428.15	0.00234	20.00	200.00	5.30
150	0.05	423.15	0.00236	20.00	200.00	5.30
145	0.06	418.15	0.00239	16.67	166.67	5.12
140	0.07	413.15	0.00242	14.29	142.86	4.96
135	0.07	408.15	0.00245	14.29	142.86	4.96
130	0.09	403.15	0.00248	11.11	111.11	4.71
125	0.1	398.15	0.00251	10.00	100.00	4.61
120	0.12	393.15	0.00254	8.33	83.33	4.42
115	0.14	388.15	0.00258	7.14	71.43	4.27
110	0.16	383.15	0.00261	6.25	62.50	4.14
105	0.19	378.15	0.00264	5.26	52.63	3.96
100	0.21	373.15	0.00268	4.76	47.62	3.86
95	0.25	368.15	0.00272	4.00	40.00	3.69
90	0.3	363.15	0.00275	3.33	33.33	3.51
85	0.36	358.15	0.00279	2.78	27.78	3.32
80	0.42	353.15	0.00283	2.38	23.81	3.17
75	0.49	348.15	0.00287	2.04	20.41	3.02
70	0.58	343.15	0.00291	1.72	17.24	2.85
65	0.71	338.15	0.00296	1.41	14.08	2.65
60	0.85	333.15	0.00300	1.18	11.76	2.47
55	1.02	328.15	0.00305	0.98	9.80	2.28
50	1.24	323.15	0.00309	0.81	8.06	2.09
45	1.53	318.15	0.00314	0.65	6.54	1.88
40	1.81	313.15	0.00319	0.55	5.52	1.71
35	2.15	308.15	0.00325	0.47	4.65	1.54
30	2.65	303.15	0.00330	0.38	3.77	1.33
25	3.14	298.15	0.00335	0.32	3.18	1.16

Table II. Source data for Fig. 5.

Current ± 1 mA	Hall Voltage (n-type) ± 0.01 V	Hall Voltage (p-type) ± 0.01 V
-59	-0.11	-0.11
-55	-0.10	-0.10
-50	-0.09	-0.09
-45	-0.08	-0.08
-40	-0.07	-0.07
-35	-0.06	-0.06
-30	-0.06	-0.06
-25	-0.05	-0.05
-20	-0.04	-0.03
-15	-0.03	-0.02
-10	-0.02	-0.02
-5	-0.01	-0.01
0	0.00	0.00
5	0.00	0.00
10	0.01	0.01
15	0.02	0.02
20	0.03	0.03
25	0.04	0.04
30	0.05	0.05
35	0.06	0.06
40	0.07	0.06
45	0.08	0.07
50	0.09	0.08
55	0.10	0.09
60	0.11	0.10

Table III. Source data for Fig. 6.

Magnetic Field Strength ±1 mT	Sample voltage (n-type) ± 0.001 V	Sample voltage (p-type) ± 0.001 V
3	1.156	1.340
20	1.156	1.339
41	1.156	1.339
61	1.157	1.340
80	1.157	1.341
100	1.157	1.343
120	1.158	1.345
140	1.159	1.347
160	1.160	1.350
180	1.161	1.353
200	1.162	1.356
220	1.164	1.359
240	1.165	1.363
260	1.167	1.366
280	1.168	1.370
300	1.170	1.375

Table IV. Source data for Fig. 7.

Magnetic Field Strength ± 1 mT	Hall Voltage (n-type) ± 0.001 V	Hall Voltage (p-type) ± 0.001 V
-300	0.065	0.061
-280	0.061	0.057
-260	0.056	0.054
-240	0.052	0.050
-220	0.048	0.046
-200	0.043	0.043
-180	0.039	0.039
-160	0.035	0.035
-140	0.030	0.031
-120	0.026	0.026
-100	0.021	0.022
-80	0.017	0.018
-60	0.013	0.013
-40	0.008	0.008
-20	0.004	0.004
0	0.001	0.000
20	-0.003	-0.003
40	-0.008	-0.008
60	-0.012	-0.012
80	-0.016	-0.016
100	-0.021	-0.021
120	-0.025	-0.025
140	-0.029	-0.029
160	-0.033	-0.033
180	-0.038	-0.037
200	-0.042	-0.041
220	-0.047	-0.045
240	-0.051	-0.049
260	-0.055	-0.053
280	-0.060	-0.057
300	-0.064	-0.060

Table V. Source data for Fig. 8.

Temperature ± 1 ^{O}C	Hall Voltage (n-type) ± 0.001 V	Hall Voltage (p-type) ± 0.001 V
170	0.002	-0.001
160	0.003	-0.002
150	0.003	-0.002
140	0.005	-0.004
130	0.008	-0.003
120	0.012	-0.002
110	0.019	0.000
100	0.026	0.004
90	0.036	0.017
80	0.046	0.031
70	0.053	0.045
60	0.059	0.054
50	0.062	0.058
40	0.063	0.059
30	0.064	0.060

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